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Unthermal Charged Massive Hawking Radiation from a Reissner-Nordström Black Hole

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Abstract Using Damour-Ruffini's method, the massive charged particles' Hawking radiation from a Reissner-Nordström black hole is investigated. When the back-reaction of particles' energy and charge to spacetime is considered, we get the unthermal spectrum. It is possible that the information will get out from the black hole with the corrected spectrum. It can be used to explain the information loss paradox, and the underlying unitary theory will be satisfied. The same conclusion as the works finished before can be drawn. However, our work is different from them, and the method is more simple and explicit.

Keywords Hawking radiation · Unitary theory · Information loss paradox · Reissner-Norström black hole

1 Introduction

In 1970s, Stephen Hawking's significant discovery that a black hole radiates thermally set up a disturbing and difficult problem about information conservation during black hole evaporation [1]. The "no hair theorem" means that when a star collapses into a black hole, it will loss all of its information except its mass M, charge Q and angular momentum J. There is no problem in classical circumstances. But when quantum effect is considered, a black hole will radiate black body spectrum, which takes nothing out from the black hole. Therefore, the information will be lost during the black hole evaporation, and this is the famous information loss paradox [2–4]. Hawking's result also breaks down the quantum mechanics because of the loss of underlying unitary theory [5–7]. If treated as a quantum state, a black hole will transform into a mixed state from a pure state, which will lead to non-unitary of the quantum gravitation. To solve this problem, Parikh and Wilczek supposed that particles

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tunnel across the event horizon. This is a semiclassical method, the barrier is created by the outgoing particles themselves, so the emission rate is calculated [8–10]. Zhang and Zhao have extended Parikh's work to more general black holes and to massive or charged particles radiation [11–15]. There are many other works about this topic [16–21], and they made this problem more and more clear. All of them can obtain the conclusion that the spectrum is no longer precise thermal and the unitary theory will be satisfied, and so an explanation to information loss paradox can be obtained.

In Liu's work [22], Hawking radiation of massive particles from Schwarzschild black hole is investigated in Damour-Ruffini's method [23]. When energy conservation and the back reaction of radiating particles to space-time are taken into account, the spectrum is also not precise thermal, the underlying unitary theory will be satisfied.

In this paper, following Liu's method, we will study the massive charged particles' Hawking radiation from Reissner-Norström black hole. We find that Liu's method can be generalized to the radiation of massive charged particles from a charged spherical symmetric black hole.

2 Klein-Gordon Equation and Tortoise Coordinate Transformation

The line element of the Reissner-Norström black hole is

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where M is the mass of the black hole, Q is the electric charge. The inner and outer horizon of the black hole are given by $r_\pm = M \pm \sqrt{M^2 - Q^2}$ respectively.

Klein-Gordon equation of charged particles in curved space-time is given by

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^\mu} - ieA_\mu\right)\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu} - ieA_\nu\right)\Phi\right] - \mu^2\Phi = 0, \tag{1}$$

where A_μ is the 4-dimensional electromagnetic potential. Since the electromagnetic field of static spherical symmetry black hole is also static and spherical symmetric, the coordinate components of A_μ have nothing to do with the coordinates t, θ, φ. Let

$$A_0 = -\frac{Q}{r}, \qquad A_1 = A_2 = A_3 = 0, \tag{2}$$

so we can write K-G equation as [24, 25]

$$\begin{aligned}
&-\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}\left[\frac{\partial^2\Phi}{\partial t^2} + \frac{2ieQ}{r}\frac{\partial\Phi}{\partial t} - \frac{e^2Q^2}{r^2}\Phi\right] \\
&\quad + \frac{2(r-M)}{r^2}\frac{\partial\Phi}{\partial r} + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\frac{\partial^2\Phi}{\partial r^2} \\
&\quad + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\Phi\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\Phi - \mu^2\Phi = 0.
\end{aligned} \tag{3}$$

By separating the variables as following

$$\Phi_{\omega lm} = N_\omega \frac{1}{r}\Psi_\omega(r,t)Y_{lm}(\theta,\varphi), \tag{4}$$

we can get the radial equation

$$\left[-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1} r^2\left(\frac{\partial^2}{\partial t^2}+\frac{2ieQ}{r}\frac{\partial}{\partial t}-\frac{e^2Q^2}{r^2}\right)+2(r-M)\frac{\partial}{\partial r} + r^2\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2}{\partial r^2}\right]\frac{\Psi_\omega}{r}-\mu^2 r^2\frac{\Psi_\omega}{r}=-l(l+1)\frac{\Psi_\omega}{r}, \tag{5}$$

and the transverse equation

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]Y_{lm}=l(l+1)Y_{lm}, \tag{6}$$

where μ, e, ω are respectively particles' mass, charge, energy, and Y_{lm} is the spherical harmonics.

Let

$$\Psi_\omega(r,t)=e^{-i\omega t}R_\omega(r), \tag{7}$$

the radial equation (5) can be written as

$$\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{\partial^2(R_\omega/r)}{\partial r^2}+\frac{2(r-M)}{r^2}\frac{\partial(R_\omega/r)}{\partial r} + \left[\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}\left(\omega^2-\frac{2eQ\omega}{r}+\frac{e^2Q^2}{r^2}\right)-\mu^2+\frac{l(l+1)}{r^2}\right](R_\omega/r)=0. \tag{8}$$

We need a coordinate system in which (8) can be written as the standard form of the wave equation at the event horizon, so we give tortoise coordinate transformation as

$$dr_*=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr, \tag{9}$$

then

$$\frac{d}{dr}=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}\frac{d}{dr_*}, \tag{10}$$

$$\frac{d^2}{dr^2}=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-2}\frac{d^2}{dr_*^2}-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-2}\left(\frac{2M}{r^2}-\frac{2Q^2}{r^3}\right)\frac{d}{dr_*}. \tag{11}$$

Substituting (9–11) into (8), we get

$$\frac{d^2}{dr_*^2}R_\omega-\left(\frac{2M}{r^2}-\frac{2Q^2}{r^3}\right)\frac{d}{dr_*}R_\omega+\frac{2(r-M)}{r^2}\frac{d}{dr_*}R_\omega + \left\{\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left[\frac{l(l+1)}{r^2}-\mu^2\right]+\omega^2-\frac{2eQ\omega}{r}+\frac{e^2Q^2}{r^2}\right\}R_\omega=0. \tag{12}$$

At the horizon $r=r_+$, (12) becomes

$$\frac{d^2}{dr_*^2}R_\omega+(\omega-\omega_0)^2R_\omega=0, \tag{13}$$

where $\omega_0=\frac{eQ}{r_+}$.

Equation (13) is the standard wave equation, its solution is

$$R_\omega = e^{\pm i(\omega-\omega_0)r_*}, \tag{14}$$

so the radial solution can be written as

$$\Psi_\omega = e^{-i\omega t \pm i(\omega-\omega_0)r_*}. \tag{15}$$

Letting $\frac{(\omega-\omega_0)}{\omega} r_* = \hat{r}$, the radial solution becomes

$$\Psi_\omega = e^{-i\omega(t\pm\hat{r})}, \tag{16}$$

where plus and minus sign are corresponding to ingoing and outgoing wave respectively.

Using advanced Eddington coordinates $v = t + \hat{r}$, we have

$$\Psi_\omega^{in} = e^{-i\omega v}, \tag{17}$$

$$\Psi_\omega^{out} = e^{2i\omega\hat{r}} e^{-i\omega v}. \tag{18}$$

The line element in the new coordinates is given by

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{19}$$

3 Analytic Extension and Back-Reaction of the Radiation

We can find that the metric (19) has no singularity at the horizon and its determinant is nonzero. Ingoing wave has good behavior at the horizon, but outgoing wave has some problems at the horizon, (18) can only describe the radiation particles outside the horizon.

To extend the wave function into the horizon, letting $(r - r_+) \to |r - r_+|e^{-i\pi} = (r_+ - r)e^{-i\pi}$, so outgoing wave function inside the horizon is

$$\Psi_\omega^{out}(r < r_+) = e^{-i\omega v}(r_+ - r)^{\frac{i}{\kappa_+}(\omega-\omega_0)} e^{\frac{\pi(\omega-\omega_0)}{\kappa_+}}. \tag{20}$$

The outgoing wave function outside the horizon is

$$\Psi_\omega^{out}(r > r_+) = e^{-i\omega v}(r - r_+)^{\frac{i}{\kappa_+}(\omega-\omega_0)}. \tag{21}$$

The emission rate of the outgoing wave function at the horizon is given by

$$\Gamma = \left|\frac{\Psi_\omega^{out}(r > r_+)}{\Psi_\omega^{out}(r < r_+)}\right|^2 = e^{\frac{-2\pi(\omega-\omega_0)}{\kappa_+}}, \tag{22}$$

where $\kappa_+ = \frac{r_+ - r_-}{2r_+^2} = \frac{\sqrt{M^2-Q^2}}{(M+\sqrt{M^2-Q^2})^2}$.

When a particle with energy ω_i and charge e_i radiates from the black hole, if the back reaction of the particle to the space-time is considered, M should be substituted by $(M - \omega_i)$, Q should be substituted by $(Q - e_i)$, the emission rate should be

$$\Gamma_i = \exp\left(\frac{-2\pi(\omega_i - \omega_{i0})}{\kappa_{i+}}\right), \tag{23}$$

where

$$\omega_{i0}=\frac{e_i(Q-e_i)}{r_{i+}}, \tag{24}$$

$$r_{i+}=(M-\omega_i)+\sqrt{(M-\omega_i)^2-(Q-e_i)^2}, \tag{25}$$

$$\kappa_{i+}=\frac{r_{i+}-r_{i-}}{2r_{i+}^2}=\frac{\sqrt{(M-\omega_i)^2-(Q-e_i)^2}}{((M-\omega_i)+\sqrt{(M-\omega_i)^2-(Q-e_i)^2})^2}. \tag{26}$$

For many particles' continuous emission, thinking that they radiate one by one, we have

$$\Gamma=\prod_i\Gamma_i=\exp\left(\sum_i\frac{-2\pi(\omega_i-\omega_{i0})}{\kappa_{i+}}\right). \tag{27}$$

If the emission is regarded as a continuous procession, the sum in (27) should be substituted by the integration. The emission rate (27) becomes

$$\Gamma=e^{\Lambda}, \tag{28}$$

where

$$\begin{aligned}
\Lambda&=-2\pi\int\frac{d\omega'+\frac{(Q-e')}{r'_+}de'}{\kappa'_+}=-2\pi\int_{(0,0)}^{(\omega,e)}\left[\frac{((M-\omega')+\sqrt{(M-\omega')^2-(Q-e')^2})^2}{\sqrt{(M-\omega')^2-(Q-e')^2}}d\omega'\right.\\
&\quad\left.-\frac{(Q-e')[(M-\omega')+\sqrt{(M-\omega')^2-(Q-e')^2}]}{\sqrt{(M-\omega')^2-(Q-e')^2}}de'\right]\\
&=-2\pi\left[\int_0^{\omega}\frac{((M-\omega')+\sqrt{(M-\omega')^2-(Q-e')^2})^2}{\sqrt{(M-\omega')^2-Q^2}}d\omega'\right.\\
&\quad\left.-\int_0^{e}\frac{(Q-e')[(M-\omega')+\sqrt{(M-\omega')^2-(Q-e')^2}]}{\sqrt{(M-\omega')^2-(Q-e')^2}}de'\right]\\
&=-\pi[((M-\omega)+\sqrt{(M-\omega)^2-(Q-e)^2})^2-(M+\sqrt{M^2-Q^2})^2]=\triangle S_{BH},
\end{aligned} \tag{29}$$

therefore, we obtain that

$$\Gamma=e^{\triangle S_{BH}}, \tag{30}$$

where in the third step of (29), we have found that the integral have nothing to do with the integral path into account, so a special path is used to simplify the calculation. $\triangle S_{BH}=S_{BH}(M-\omega,Q-e)-S_{BH}(M,Q)$, and $\triangle S_{BH}$ is the difference between the entropy of the black hole before and after the emission.

4 Discussions

The emission rate (30) is consistent with the underlying unitary theory, and it takes the same form as previous results. Moreover, (30) also implies that the radiation spectrum is not purely thermal. To compare with the thermal spectrum, we expand $\triangle S_{BH}$ in ω and e. After higher-order terms of ω and e are neglected, we have

$$\triangle S_{BH} = -\beta(\omega - \omega_0),$$

where $\omega_0 = \frac{Qe}{M+\sqrt{M^2-Q^2}}$, $\beta = \frac{2\pi(M+\sqrt{M^2-Q^2})^2}{\sqrt{M^2-Q^2}}$, and β is the inverse temperature. Some information can be taken out from the black hole with the corrected spectrum (30). It can be used to explain the information loss paradox in black hole.

We calculated the massive charged particles' Hawking radiation from a Reissner-Norström black hole in Damour-Ruffini's method, which has become a successful method to study Hawking radiation. In this method, using the relativistic quantum mechanics in curved space time, we can study not only the static and the stationary black holes, but also the dynamical ones. Furthermore, the radiation of many kinds of particles can be studied in this way such as bosons, fermions etc. When energy conservation, charge conservation and back-reaction of radiation to space-time are taken into account, we obtained a possible explanation to information loss paradox. This method is not only more simple than previous works, but also has an explicit physics picture.

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References

1. Hawking, S.W.: Commun. Math. Phys. **43**(3), 199 (1975)
2. Hawking, S.W.: Phys. Rev. D **72**, 084013 (2005)
3. Preskill, J.: Do black holes destroy information? Arxiv: hep-th/9209058 (1992)
4. Zhang, J.-Y., Hu, Y.-P., Zhao, Z.: Mod. Phys. Lett. A **21**,1865 (2006)
5. Hawking, S.W., Penrose, R.: The Nature of Space and Time. Princeton University Press, Princeton (1996)
6. Hawking, S.W.: Nature **248**, 30 (1974)
7. Hawking, S.W.: In: 17th Int. Conf. on GRG, Dubin, 21 July 2004
8. Parikh, M.K.: Int. J. Mod. Phys. D **13**, 2355 (2004)
9. Parikh, M.K.: Energy conservation and Hawking radiation. Arxiv:hep-th/0402166 (2004)
10. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
11. Zhang, J.-Y., Zhao, Z.: Phys. Lett. B **638**, 110 (2006)
12. Zhang, J.-Y., Zhao, Z.: J. High Energy Phys. **10**, 055 (2005)
13. Zhang, J.-Y., Zhao, Z.: Phys. Lett. B **618**, 14 (2005)
14. Zhang, J.-Y., Zhao, Z.: Mod. Phys. Lett. A **20**, 2449 (2005)
15. Zhang, J.-Y., Zhao, Z.: Nucl. Phys. B **725**, 173 (2005)
16. Ren, J., Zhang, J.-Y., Zhao, Z.: Chin. Phys. Lett. **23**, 2019 (2006)
17. Hu, Y.-P., Zhang, J.-Y., Zhao, Z.: Mod. Phys. Lett. A **21**, 2143 (2006)
18. Gao, L., Liu, W.-B.: Commun. Theor. Phys. **46** (2006)
19. Gao, L., Zhang, H.-B., Liu, W.-B.: Int. J. Theor. Phys. **46**, 33 (2007)
20. Fang, H.-Z., Hu, Y.-P., Zhao, Z.: Chin. Phys. Lett. **22**, 1611 (2005)
21. Fang, H.-Z., Ren, J., Zhao, Z.: Int. J. Mod. Phys. D **14**, 1699 (2005)
22. Liu, W.-B.: J. Beijing Norm. Univ. (Nat. Sci.) **43**, 144 (2007)
23. Damour, T., Ruffini, R.: Phys. Rev. D **14**, 332 (1976)
24. Liang, C.-B., Zhou, B.: Differential Geometry and General Relativity. Science Press, Beijing (2006)
25. Zhao, Z.: The Thermal Nature of Black Holes and the Singularity of the Space-Time. Beijing Normal University Press, Beijing (1999)